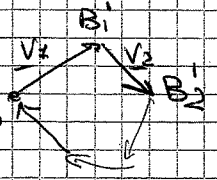
SOMMA: n vettori qualsiasi $\underline{v}_1, \underline{v}_2, \underline{v}_n$

Scelto punto O dello spazio e presidi i vettori equipollenti a $\underline{v}_1$ tali $O, \underline{v}_2'$ equipollente a $\underline{v}_2$ con origine in B_1'



$\underline{v}_1 + \underline{v}_2 + \dots + \underline{v}_n = \overrightarrow{OB'_n} = \underline{R} = \sum_{i=1}^{n} \underline{v}_i$

se $B'_n \equiv O \rightarrow \underline{R} = 0$

ASSOCIATIVA, COMMUTATIVA, REGOLA PARALLELOGRAMMA

DECOMPOSIZIONE $\underline{v} = \overrightarrow{AB}$

Si può decomporre $\overrightarrow{AB}$ in infiniti modi in modo che somma sia $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AA_1} + \overrightarrow{A_1A_2} + \dots + \overrightarrow{A_{n-1}B}$

- 2 RETTE COMPLANARI AL VETTORE DATO $\underline{v}_1 + \underline{v}_2 = \underline{v}$ ($\underline{v}_1 \nparallel r_1$, $\underline{v}_2 \nparallel r_2$)
- 3 RETTE NON COMPLANARI $\overrightarrow{OA_1} + \overrightarrow{A_1A_2} + \overrightarrow{A_2A} = \underline{v}$ ($\overrightarrow{OA_1} \nparallel r_1$, $\overrightarrow{A_1A_2} \nparallel r_2$, $\overrightarrow{A_2A} \nparallel r_3$)
(non è unica)
- UNA RETTA r e UN PIANO $\perp$ A r

$\overrightarrow{OA_1} + \overrightarrow{OA_2} = \overrightarrow{OP} = \underline{v}$

$\overrightarrow{OA_1} \parallel r$, $\overrightarrow{OA_2} \nparallel r$

IL COMPONENTE → DIREZIONE

IL COMPONENTE → SCALARE

PRODOTTO PER UNO SCALARE

$\underline{u} = m\underline{v}$ $\quad |\underline{u}| = |m|\,|\underline{v}|$ $\quad m = \pm \frac{|\underline{u}|}{|\underline{v}|}$

$\rightarrow m(n\underline{v}) = n(m\underline{v}) = (mn)\underline{v}$

$\rightarrow m(\underline{v} + \underline{u}) = m\underline{v} + m\underline{u}$

$\rightarrow (m+n)\underline{v} = m\underline{v} + n\underline{v}$

$\underline{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \underline{v}_x + \underline{v}_y + \underline{v}_z$

PRODOTTO SCALARE

$\boxed{\underline{u} \cdot \underline{v} = |\underline{u}|\,|\underline{v}| \cos\varphi}$

$\underline{u} = |\underline{v}| \cos\varphi$ PROIEZIONE DI $\underline{v}$ SU $\underline{u}$

$\underline{v} = |\underline{u}| \cos\varphi$ PROIEZIONE DI $\underline{u}$ SU $\underline{v}$

$\rightarrow \boxed{\underline{u} \cdot \underline{v} = 0 \iff \underline{u} \perp \underline{v}}$

$\rightarrow \underline{u} \cdot \underline{v} = \underline{v} \cdot \underline{u}$

$\rightarrow \underline{u} \cdot (\underline{v}_1 + \underline{v}_2) = \underline{u}\underline{v}_1 + \underline{u}\underline{v}_2$

$\rightarrow \underline{u} \cdot \underline{u} = 0 \Rightarrow \underline{u} = 0$

$\rightarrow (m\underline{u}) \cdot \underline{v} = \underline{u}(m\underline{v}) = (\underline{u} \cdot \underline{v})m$

$\rightarrow \underline{v} \cdot \underline{v} = |\underline{v}|^2$

$\rightarrow \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1 \qquad \vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{k} = 0$

DELTA DI KRONECKER

$\vec{i}_h \cdot \vec{i}_k = \delta_{hk}$ $\begin{cases} 1 \text{ se } h=k \\ 0 \text{ se } h \neq k \end{cases}$

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$\underline{u} \cdot \underline{v} = (u_x \hat{i} + u_y \hat{j} + u_z \hat{k}) \cdot (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) =$

$= u_x v_x (\hat{i} \cdot \hat{i}) + u_y v_y (\hat{j} \cdot \hat{j}) + u_z v_z (\hat{k} \cdot \hat{k}) = u_x v_x + u_y v_y + u_z v_z$

se $\underline{u} = u_h \hat{i}_h \qquad \underline{v} = v_k \hat{i}_k$

$\underline{u} \cdot \underline{v} = (u_h \hat{i}_h) \cdot (v_k \hat{i}_k) = u_h v_k (\hat{i}_h \cdot \hat{i}_k) = \boxed{u_h v_k \delta_{hk}}$

$= u_h v_h = u_1 v_1 + u_2 v_2 + u_3 v_3$

CONVENZIONE DI EINSTEIN (SUGLI INDICI RIPETUTI) $\sum_{h=1}^{3} u_h \hat{i}_h = u_h \hat{i}_h$

PRODOTTO VETTORIALE

$\boxed{\underline{u} \wedge \underline{w} = |u| \cdot |v| \sin\varphi}$

$|\underline{w}| = |u| \cdot |v| \sin\varphi$ = AREA PARALLELOGRAMMA

$\underline{w} \perp \underline{u}$, $\underline{w} \perp \underline{v}$, $\underline{w}$ ⊥ PIANO DI $\underline{u}$ e $\underline{v}$ → REGOLA DESTRORSA

$\boxed{\underline{u} \wedge \underline{v} = 0 \iff u \parallel v}$

→ $\underline{u} \wedge \underline{v} = -\underline{v} \wedge \underline{u}$ → $(u \wedge v) \wedge w \neq u \wedge (v \wedge w)$

→ $\underline{u} \wedge (\underline{v}_1 + \underline{v}_2) = \underline{u} \wedge \underline{v}_1 + \underline{u} \wedge \underline{v}_2$

→ $m (\underline{u} \wedge \underline{v}) = (m\underline{u}) \wedge \underline{v} = \underline{u} \wedge (m\underline{v})$

→ $\hat{i} \wedge \hat{i} = \hat{j} \wedge \hat{j} = \hat{k} \wedge \hat{k} = 0 \qquad \hat{i} \wedge \hat{j} = \hat{k}, \ \hat{j} \wedge \hat{k} = \hat{i} \qquad \hat{k} \wedge \hat{i} = \hat{j}$

$\underline{u} \wedge \underline{v} = (u_x \hat{i} + u_y \hat{j} + u_z \hat{k}) \wedge (v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) =$

$= (u_y v_z - u_z v_y) \hat{i} + (u_z v_x - u_x v_z) \hat{j} + (u_x v_y - u_y v_x) \hat{k}$

$\varepsilon_{ijk} = \begin{cases} 1 & \text{se } i,j,k \text{ sono una permutazione pari } (123, 231, 312) \\ -1 & \text{se } i,j,k \text{ sono una permutazione dispari } (132, 213, 321) \\ 0 & \text{se } ci \text{ sono } 2 \text{ indici uguali} \end{cases}$

$A_{ik} = \varepsilon_{ijk} u_j = \begin{pmatrix} i=k=1 & i=1 \ k=2 \ j=3 & i=1 \ k=3 \ j=2 \\ i=2 \ k=1 \ j=3 & i=k=2 & i=2 \ k=3 \ j=1 \\ i=3 \ k=1 \ j=2 & i=3 \ k=2 \ j=1 & i=k=3 \end{pmatrix} = \begin{pmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{pmatrix}$

$\boxed{(\underline{u} \wedge \underline{v})_i = \varepsilon_{ijk} u_j v_k}$

$\varepsilon_{ijk} \varepsilon_{lmn} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}$

PRODOTTO MISTO

$$\underline{u} \cdot (\underline{v} \wedge \underline{w}) = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

$\underline{u} \cdot (\underline{v} \wedge \underline{w}) = 0 \iff \underline{u}, \underline{v}, \underline{w}$ COMPLANARI

$$\underline{u} \cdot (\underline{v} \wedge \underline{w}) = \underline{w} \cdot (\underline{u} \wedge \underline{v}) = \underline{v} \cdot (\underline{w} \wedge \underline{u})$$
$$= -\underline{u} \cdot (\underline{w} \wedge \underline{v}) = -\underline{w} \cdot (\underline{v} \wedge \underline{u}) = -\underline{v} \cdot (\underline{u} \wedge \underline{w})$$

$$(\underline{v} \wedge \underline{w})_i = \varepsilon_{ijk} v_j w_k$$
$$(\underline{a} \cdot \underline{b}) = a_h b_k \delta_{hk}$$

$$u_e (\varepsilon_{ijk} v_j w_k) \delta_{ie} = u_e \varepsilon_{ijk} v_j w_k = \varepsilon_{eik} u_e v_j w_k$$

DOPPIO PRODOTTO VETTORIALE

$$(\underline{u} \wedge \underline{v}) \wedge \underline{w} = (\underline{w} \cdot \underline{u}) \underline{v} - (\underline{v} \cdot \underline{w}) \underline{u}$$

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CURVE NEL PIANO E NELLO SPAZIO

VETTORI DIPENDENTI DAL TEMPO

$\underline{V}(t) = V_x(t)\,\hat{i} + V_y(t)\,\hat{j} + V_z(t)\,\hat{k}$

SE LE 3 FUNZIONI SONO CONTINUE $\Rightarrow \underline{V}(t)$ È CONTINUA

$\lim_{t \to t_0} \underline{V}(t) = \underline{V}_0(t_0)$ CONTINUITÀ

DERIVATA

$$\lim_{\Delta t \to 0} \frac{\underline{V}(t+\Delta t) - \underline{V}(t)}{\Delta t} = \underline{V}'(t) = \frac{d\underline{V}}{dt}$$

$\rightarrow V_z = \underline{V} \cdot \hat{z} \qquad V_z(t) = \underline{V}(t) \cdot \hat{z}$

$$\frac{dV_z}{dt} = \frac{d\underline{V}}{dt} \cdot \hat{z}$$

$$\frac{dV_z}{dt} = \lim_{\Delta t \to 0} \frac{V_z(t+\Delta t) - V_z(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\underline{V}(t+\Delta t)\cdot\hat{z} - \underline{V}(t)\cdot\hat{z}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{[\underline{V}(t+\Delta t) - \underline{V}(t)]\,\hat{z}}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{\underline{V}(t+\Delta t) - \underline{V}(t)}{\Delta t}\right) \cdot \hat{z}$$

$$= \frac{d\underline{V}}{dt} \cdot \hat{z}$$

$\rightarrow \frac{d\underline{V}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$

$\rightarrow \frac{d}{dt}\left[f(t)\,\underline{V}(t)\right] = \frac{df}{dt}\,\underline{V}(t) + f(t)\,\frac{d\underline{V}}{dt}$

$\rightarrow \frac{d}{dt}\left(\underline{u}\cdot\underline{v}\right) = \frac{d\underline{u}}{dt}\cdot\underline{v} + \underline{u}\cdot\frac{d\underline{v}}{dt}$

$\rightarrow \frac{d}{dt}\left(\underline{u}\wedge\underline{v}\right) = \frac{d\underline{u}}{dt}\wedge\underline{v} + \underline{u}\wedge\frac{d\underline{u}}{dt}$

$\rightarrow$ SE $|\underline{V}| = \cos t \Rightarrow \frac{d\underline{V}}{dt} \perp \underline{V}$

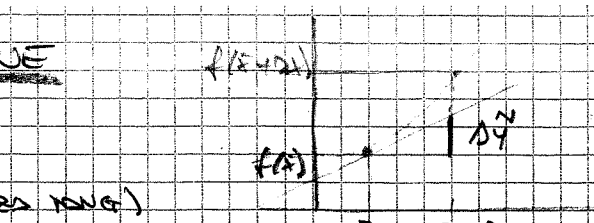
$\underline{V}\cdot\underline{V} = \cos t$

$$0 = \frac{d}{dt}(\underline{V}\cdot\underline{V}) = \frac{d\underline{V}}{dt}\cdot\underline{V} + \underline{V}\cdot\frac{d\underline{V}}{dt} = 2\,\underline{V}\cdot\frac{d\underline{V}}{dt}$$

$\underline{V}\cdot\frac{d\underline{V}}{dt} = 0 \Rightarrow \frac{d\underline{V}}{dt} \perp \underline{V}$

SE $\underline{V}(t(s)) \Rightarrow \frac{d\underline{V}}{dt} = \frac{d\underline{V}}{ds}\cdot\frac{ds}{dt}$

DIFFERENZIALE FUNZIONE



$\Delta y = f(x+\Delta x) - f(x)$

$\Delta \tilde{y} = f'(x) \cdot \Delta x$ (RETTA)

$\Delta y - \Delta \tilde{y} = f(x+\Delta x) - f(x) - f'(x)\Delta x$

$df = f'(x)\,dx$

$\Delta f \cong f'(x)\,\Delta x$ OPPORTUNE CHE DA OTTENERE Δf È LINEARE IN Δx

$\frac{dV}{dt} = \left(\frac{dV}{dt}\right) dt \qquad \Delta V = \frac{dV}{dt}\,\Delta t$

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LUNGHEZZA CURVA

$OP = f_1(N)\,\hat{i} + f_2(N)\,\hat{j} + f_3(N)\,\hat{k}$

$|AB|^2 = |OB - OA|^2 = \left|\left(f_1(N_B)\hat{i} + f_2(N_B)\hat{j} + f_3(N_B)\hat{k}\right) - \left(f_1(N_A)\hat{i} + f_2(N_A)\hat{j} + f_3(N_A)\hat{k}\right)\right|^2 =$

$= \left|\left(f_1(N_B) - f_1(N_A)\right)\hat{i} + \left(f_2(N_B) - f_2(N_A)\right)\hat{j} + \left(f_3(N_B) - f_3(N_A)\right)\hat{k}\right|^2 =$

$= \left(f_1(N_B) - f_1(N_A)\right)^2 + \left(f_2(N_B) - f_2(N_A)\right)^2 + \left(f_3(N_B) - f_3(N_A)\right)^2$

$= \left[\left(\frac{df_1}{dN}\Big|_{N_A} \cdot \Delta N\right)^2 + \left(\frac{df_2}{dN}\Big|_{N_A} \cdot \Delta N\right)^2 + \left(\frac{df_3}{dN}\Big|_{N_A} \cdot \Delta N\right)^2\right]$

$= (\Delta N)^2 \left[\left(\frac{df_1}{dN}\right)^2 + \left(\frac{df_2}{dN}\right)^2 + \left(\frac{df_3}{dN}\right)^2\right] = |AB|^2$

$|AB| \cong \sqrt{\left(\frac{df_1}{dN}\right)^2 + \left(\frac{df_2}{dN}\right)^2 + \left(\frac{df_3}{dN}\right)^2}\,dN$

$$L = \int_{N_1}^{N_2} \sqrt{\left(\frac{df_1}{dN}\right)^2 + \left(\frac{df_2}{dN}\right)^2 + \left(\frac{df_3}{dN}\right)^2}\,dN$$

$$ds = \sqrt{\left(\frac{df_1}{dN}\right)^2 + \left(\frac{df_2}{dN}\right)^2 + \left(\frac{df_3}{dN}\right)^2}\,dN$$

VERSORE TANGENTE

$N_B = N_A + \Delta N$

$\vec{T} = T_x\hat{i} + T_y\hat{j} + T_z\hat{k}$

$\lim_{\Delta N \to 0} \frac{f_1(N_A + \Delta N) - f_1(N_A)}{\Delta N} = \frac{df_1}{dN}\Big|_{N_A}$

$\vec{T} = \frac{df_1}{dN}\Big|_{N_A}\hat{i} + \frac{df_2}{dN}\Big|_{N_A}\hat{j} + \frac{df_3}{dN}\Big|_{N_A}\hat{k}$

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|} = \frac{\frac{df_1}{dN}\hat{i} + \frac{df_2}{dN}\hat{j} + \frac{df_3}{dN}\hat{k}}{\sqrt{\left(\frac{df_1}{dN}\right)^2 + \left(\frac{df_2}{dN}\right)^2 + \left(\frac{df_3}{dN}\right)^2}} = \frac{dOP}{dS}$$